AP Calculus AB

WS 77 Mean Value Theorem

1) $f(x) = x^3 + 1$; $[2, 3]$

$$\frac{f(3) - f(2)}{3-2} = \frac{28 - 9}{1}$$

$$= 19$$

2) $f(x) = \sqrt{4x+1}$ on $[0, 2]$

$$ROC_{avg} = \frac{f(2) - f(0)}{2-0}$$

$$= \frac{3-1}{2}$$

$$= 1$$

3) $f(x) = e^x$; on $[-2, 0]$

$$ROC_{avg} = \frac{f(0) - f(-2)}{0+2}$$

$$= \frac{1 - e^{-2}}{2}$$

$$= \frac{1 - \frac{1}{e^2}}{2}$$

$$= \frac{e^2 - 1}{2e^2}$$

4) $f(x) = \ln x$ on $[100, 103]$

$$ROC_{avg} = \frac{f(103) - f(100)}{3}$$

$$= \frac{\ln 103 - \ln 100}{3}$$

$$= \frac{\ln\left(\frac{103}{100}\right)}{3}$$

5) $f(x) = \sin x$ on $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

$$ROC_{avg} = \frac{f\left(\frac{3\pi}{4}\right) - f\left(\frac{\pi}{4}\right)}{\frac{3\pi}{4} - \frac{\pi}{4}}$$

$$= 0$$

6) $f(x)$ is cont. & diff. $f(x) = x^2 - 2x - 2$

$$f'(x) = \frac{f(3) - f(-1)}{4}$$

$$2x - 2 = \frac{1 - 1}{4}$$

$$2x - 2 = 0$$

$$\boxed{x = 1}$$

7) $f(x) = x^3 - x$ on $[0, 1]$

$$f'(x) = \frac{f(1) - f(0)}{1-0}$$

$$3x^2 - 1 = 0$$

$$3x^2 = 1$$

$$x^2 = 1/3$$

$$\boxed{x = 1/\sqrt{3}}$$

8) $f(x) = \frac{x^2 - x - 6}{x - 1}$ on $[-2, 3]$

$f(x)$ is not cont @ $x = 1$

MVT does not apply.

9) $f(x) = \sin x$ on $[0, \pi]$

$f(x)$ is cont. & diff.

$$f'(x) = \frac{f(\pi) - f(0)}{\pi}$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}$$

10) $f(x) = \tan x$ on $(0, \pi)$

$f(x)$ is not cont. @ $x = \frac{\pi}{2}$

MVT does not apply

11) $f(x) = x^2 - 2x + 1$ on $[1,3]$

$f(x)$ is cont. & diff.

$f'(x) = \frac{f(3) - f(1)}{3-1}$

$2x - 2 = \frac{4-0}{2}$

$2x - 2 = 2$

$\boxed{x = 2}$

12) $f(x) = x^{3/4}$ on $[0,16]$

$f(x)$ is cont. & diff.

$f'(x) = \frac{3}{4}x^{-1/4}$

$\frac{3}{4x^{1/4}} = \frac{8}{16} = \frac{1}{2}$

$6 = 4x^{1/4}$

$\frac{3}{2} = x^{1/4}$

$\boxed{x = \frac{81}{16}}$

13) $f(x) = \sqrt{1 - x^2}$ on $[-1,1]$

$f(x)$ is cont on $[-1,1]$

$f(x)$ is diff on $(-1,1)$

$f'(x) = \frac{f(1) - f(-1)}{1 - (-1)}$

$\frac{1}{2}(1-x^2)^{-1/2} \cdot (-2x) = 0$

$\frac{-x}{\sqrt{1-x^2}} = 0$

$x = 0$

14) $f(x) = \frac{1}{x^2}$ on $[-1,1]$

$f(x)$ is not cont @ $x = 0$

MVT does not apply.

2007 AP Question

a) $h(x) = f(g(x)) - 6$

$h(r) = -5$? Use IVT

$h(x)$ is cont. on $[1,3]$

$h(1) = f(g(1)) - 6 = f(2) - 6$

$= 3 > -5$ ✓

$h(3) = f(g(3)) - 6 = f(4) - 6$

$= -7 < -5$

By IVT $h(r) = -5$ in $(1,3)$

b) $h'(c) = -5$?

$h'(x) = \frac{h(3) - h(1)}{3 - 1}$

$= \frac{-7 - 3}{2}$

$= -5$

By MVT $h'(c) = -5$ ✓